

# CS 70 Challenge Problems:

## Basic Probability

Solutions at <https://alextseng.net/teaching/cs70/>  
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### 1 General Probability

- (a) If 150 fair coins are tossed in the air, what is the probability that exactly 40 of them land up heads?
- (b) Recall the Monty Hall problem. Consider a variant of the game where the contestant is faced with 5 doors. He/she selects a single door, and the host reveals the contents of one of the 4 other doors (which turns out to not have the prize). What is the probability the contestant will win the prize if he/she stays with the current door? What is the probability of winning if he/she switches? What is the best strategy?
- (c) Carol and Dave each have an identical bag of 5 marbles. Each bag contains a red, orange, yellow, green, blue, and purple marble. Carol reaches into her bag and randomly picks out a marble and puts it into Dave's bag. Dave reaches into his bag and randomly picks out a marble and puts it into Carol's bag. What is the probability that after these 2 actions, the bags each have 5 distinctly colored marbles again?

(d) \*Challenge\* Alice selects 3 random numbers from the set  $\{1, \dots, 9\}$ , and places them in descending order to form a 3-digit number. Bob does the same thing, but selects 3 random numbers from the set  $\{1, \dots, 8\}$ . Both Alice and Bob select without replacement. What is the probability that Alice's 3-digit number is larger than Bob's 3-digit number?

(e) \*Challenge\* Consider an unfair pair of dice. The two dice are identical, and each has probabilities of rolling 1, 2, 3, 4, 5, and 6, in a ratio of 1:2:3:4:5:6 (e.g. it is six times more likely to roll a 6 than a 1). Upon rolling these two dice, what is the probability of rolling a total of 7?