

# CS 70 Challenge Problems: Conditional Probability

Solutions at <https://alextseng.net/teaching/cs70/>  
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## 1 Conditional Probability

- (a) Consider 3 cards. One card is blue on both sides, one card is gold on both sides, and one card is blue on one side and gold on the other side. A single card is randomly and placed on a table. One side of the card is blue. What is the probability the other side of the card is also blue?

## 2 Bayes' Rule

- (a) Due to your many hours of diligent studying for the next CS 70 midterm, you have successfully learned 80% of the material. That is, on the exam, each question has an 80% chance of being one which you know the correct answer to. On the questions you don't know the answer to, however, you can still guess. Since all of the questions are multiple choice, you will have a 20% chance of getting a question right if you guess. A week later, after the test is over, you decide to look through the graded exam and see what you still need to review. For any question, given that you got the correct answer, what is the probability that you actually knew the answer (rather than just guessing)?  
Hint: It might help to first calculate the probability that you will give the correct answer to any question.

- (b) A robot is trying to cross the road safely. It has a sensor to help it determine the road's conditions, but because of imperfect hardware and changing conditions, it may not always give correct information. The robot knows that if the conditions are truly safe, the sensor will correctly tell it so with  $\frac{5}{6}$  probability. But if the conditions are unsafe, the sensor may still tell the robot that it is safe with  $\frac{1}{10}$  probability. The robot also knows that in general, the road is safe to cross about  $\frac{1}{3}$  of the time. Given that the robot's sensor indicates that it is safe to cross, what is the probability it actually is safe to cross?

### 3 Independence

- (a) 2 cards are chosen randomly from a deck of card with replacement (after drawing the first card, it is replaced and the deck is shuffled before drawing the second). Given the first card was a spade, what is the probability that the second card is also a spade?
- (b) Consider rolling 2 dice. Let  $A$  be the event that the first die comes up as an odd number. Let  $B$  be the event that the second die comes up as an odd number. Let  $C$  be the event that the sum of the dice values is odd. Intuitively, are  $A, B, C$  pairwise independent? That is, are any pair of these events independent? What are the probabilities of each event? Of each pair?
- (c) Continuing from part (b), are the events  $A, B, C$  mutually independent? That is, is each one independent from the other two? What is the probability of all 3 of them happening together?