CS 70 Challenge Problems: Distributions

Solutions at https://alextseng.net/teaching/cs70/

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1 Distributions to Use

Find the distribution (discrete or continuous) that is most appropriate for each of these scenarios. Then, answer the question using the specified distribution.
Possible distributions are uniform, binomial, Poisson, geometric, exponential, and Gaussian

(a) Every day, 1000 people play the lottery. Each person buys a ticket with randomized numbers. The chance of winning is $\frac{1}{10000}$. What is the probability that fewer than 3 people win?

(b) Due to human impact on climate and habitats, fewer and fewer Roan antelope can be seen in South Africa's Kruger National Park. If there are an average of 2 sightings of Roan antelope per day, what is the probability that there are 7 sightings in a week?

(c) A card is drawn at random from a 52-card deck. What is the probability that the card drawn is a face card (i.e. jack, queen, or king)?

(d) On average, there are 2 car crashes per hour in the county. How many minutes are expected until the next car crash?

(e) The average score on a recent exam was 70%. Only 10 students scored above 90%. How many students are expected to have scored below 50%?

(f) Steven the basketball player is trying to make 3-point shots. The probability he can make a 3-point shot is always $\frac{1}{10}$. If each shot takes 20 seconds total, how long do we expect to wait until he finally successfully shoots one into the basket?

2 Combining Distributions

(a) A company's server uses 2 hard drives—one for normal use and the other for backup. Both hard drives are identical, and each is expected to fail in 4 years. As soon as the first hard drive fails, the company will start using the second one (which had not been used until now). What is the expected time until both hard drives fail? What is the variance?

(b) *Challenge* Given two independent Poisson random variables X and Y with means λ_x and λ_y, respectively, show that $X + Y$ is also a Poisson random variable. What is its mean?

3 Central Limit Theorem

z-score table:

z	$P(X \leq z)$ where $X \sim N(0, 1)$
2.5	0.9938
3.5	0.9999

(a) You buy 100 identical lightbulbs from the hardware store. The manufacturer of these lightbulbs promises that on average, they have a lifespan of 2 years. Assume that these 100 lightbulbs are a random sample from the factory. You start using these 100 lightbulbs and record how long it takes each one to fail. Let T be the average time it took to fail (over all 100 lightbulbs). What is the probability that $T \leq 1.5$? Use the provided z-score table.

(b) Hoping for better performance, you want the probability that $T \leq 1.5$ to be smaller. How many lightbulbs would you need to buy in order for T, the average lifespan, to be no more than 0.0001?