CS 70 Challenge Problems:
Proof Techniques

Solutions at https://alextseng.net/teaching/cs70/

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1 Proofs

Prove the following propositions using a direct proof, proof by contradiction, contraposition, or a proof by cases.

(a) $(\forall x, y \in \mathbb{Z})\ 6 \nmid xy \Rightarrow (6 \nmid x \wedge 6 \nmid y)$
In plain English: For all integers x and y, if xy is not divisible by 6, then neither x nor y are divisible by 6.

(b) Every integer that is a perfect cube is either a multiple of 9, 1 more than a multiple of 9, or 1 less than a multiple of 9.

(c) The difference between a rational number and an irrational number is irrational.

(d) *Challenge* $\sqrt{2}$ is irrational.

2 Induction

(a) Assume that $P(x) \Rightarrow P(x+2)$. What would you need to show in order to prove that $P(x)\ \forall x \in \mathbb{N}$?

(b) Is this proof correct? If not, explain why.
$\forall n \in \mathbb{N}\ (42^n = 1)$
Base Case: $n = 0$, $42^0 = 1$
Inductive Hypothesis: Assume that $42^k = 1$.
Inductive Step: $42^{k+1} = \frac{42^k \times 42^k}{42^{k-1}} = \frac{1 \times 1}{1} = 1$.