CS 70 Challenge Problems:
Stable Marriage

Solutions at https://alextseng.net/teaching/cs70/

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1 Stable Marriage Properties

(a) Recall that the optimal man for a woman is her most preferred man that she may be paired with, across all possible stable pairings. True or False: two women may have the same optimal man.

(b) True or False: In executing the Stable Marriage Algorithm, if the algorithm lasts n days, then there exists a woman who is not proposed to on day $n-3$, assuming $n > 3$.

(c) True or False: Upon running the Stable Marriage Algorithm, it is possible that every man gets his last choice.

(d) For a normal run of the Stable Marriage Algorithm, let $P_i(M)$ be the rank of the woman that M proposes to on day i. Let $R_i(W)$ be the number of the men that W has rejected so far up to i, not including any rejections on day i. For any day i, what is the value of $\sum_M P_i(M) - \sum_W R_i(W)$?

(e) Consider an instance of the Stable Marriage Problem with n men and n women. In this instance, there are exactly three stable pairings possible: M_1, M_2, M_3. Every woman is matched to a different man in each of the three matchings, so every woman has a clear ranking of which matching she would prefer (according to her preference list). It turns out that some woman W prefers M_3 to M_2 and M_2 to M_1 ($M_3 > M_2 > M_1$). True or False: every woman must have the same ranking for the matchings $M_3 > M_2 > M_1$.